

## Properties of Functions

A function is a mathematical relation such that for every input (usually  $x$ ) there is at most one output (usually  $y$ .)

The following properties (operations) hold true for all functions:

Addition of Functions	$(f + g)(x) = f(x) + g(x)$
Subtraction of Functions	$(f - g)(x) = f(x) - g(x)$
Multiplication of Functions	$(f \cdot g)(x) = f(x) \cdot g(x)$
Division of Functions	$\left(\frac{f}{g}\right)(x) = (f \div g)(x) = f(x) \div g(x)$ This is true as long as $g(x) \neq 0$
Composition of Functions*	$(f \circ g)(x) = f(g(x))$ $(g \circ f)(x) = g(f(x))$ If $(f \circ g)(x) = (g \circ f)(x) = x$ then the functions are inverses of each other

If  $f(-x) = f(x)$  then the function is even and symmetric over the  $y$ -axis.

If  $f(-x) = -f(x)$  then the function is odd and is symmetric about the origin.

A function is either even, odd, or neither. A function can not be both even and odd.

\*The domain of  $(f \circ g)(x)$  is the set of  $x$  in the domain of  $g(x)$  such that  $g(x)$  is in the domain of  $f$ .

Think about the domain of composite functions like this:

If you have  $f(g(x))$  this means you plug a number,  $x$ , into  $g(x)$ . You then take this output and then put it into  $f(x)$ . Your domain is then any  $x$  that when put into  $g(x)$  will give you something that you can then put into  $f(x)$