Binomial Theorem

In the expansion of $(x+y)^n$

$$(x+y)^n = x^n + nx^{n-1}y + ... + {}_nC_m x^{n-m}y^m + ... + nxy^{n-1} + y^n$$

Where:

$${}_nC_m = \frac{n!}{(n-m)!m!}$$

Example:

$$(x+y)^0 = 1, \ \ x+y \neq 0$$

$$(x+y)^1 = x+y$$

$$(x+y)^2 = x+2xy+y$$

$$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$$

$$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$$

5 Things about all expanded binomials:

1. There are always n+1 terms
2. The powers of x decrease by 1, powers of y increase by 1
3. The sum of the powers in each term is n
4. First term is x^n and the last term is y^n and both have coefficients of 1
5. Coefficients increase and decrease in a symmetrical pattern.

If the two terms are addition, all of the terms in the expansion are positive. If the terms are subtracted, the signs of the terms alternate.

Example:

$$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$$

$$(x-y)^4 = x^4-4x^3y+6x^2y^2-4xy^3+y^4$$

Pascal's Triangle

A triangle of coefficients for each term for the expansion of a binomial as shown below. Each number is the sum of the two numbers above it. The first and last numbers in each row are always 1.

Binomial Theorem Notes

1								Row 0
1	1							Row 1
1	2	1						Row 2
1	3	3	1					Row 3
1	4	6	4	1				Row 4
1	5	10	10	5	1			Row 5
1	6	15	20	15	6	1		Row 6
1	7	21	35	35	21	7	1	Row 7

Note that the top row of the triangle is "row 0." This is because it is the row used when a binomial is raised to the 0 power. Also, note that the second term of the row (with the exception of row 0) is also the name of the row.

<u>Example</u>:

$(2a+3b)^4$

Use the forth row of Pascal's Triangle (underlined in the problem below):

1	4	6	4	1

$(2a+3b)^4$ compared to $(x+y)^4$

$x=2a$ and $y=3b$

$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$

$(2a+3b)^4 = \underline{1}(2a)^4+\underline{4}(2a)^3(3b)+\underline{6}(2a)^2(3b)^2+\underline{4}(2a)(3b)^3+\underline{1}(3b)^4$

$(2a+3b)^4 = \underline{1}\left(2^4a^4\right)+\underline{4}\left(2^3a^3\right)(3b)+\underline{6}\left(2^2a^2\right)\left(3^2b^2\right)+\underline{4}(2a)\left(3^3b^3\right)+\underline{1}\left(3^4b^4\right)$

$(2a+3b)^4 = 16a^4+96a^3b+216a^2b^2+216ab^3+81b^4$

Binomial Theorem Notes

Example:

$(2a-3b)^4$ compared to $(x-y)^4$

$x=2a$ and $y=3b$

Continue as above, bote that y does NOT equal -3b, the signs go in at the end.

The final answer is:

$(2a+3b)^4 = 16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4$

