Counting Principles

Fundamental Counting Principle

-Let E_1 and E_2 be two different events. The first event E_1 can occur in m_1 different ways. After E_1 has occurred, E_2 can occur in m_2 different ways. The number of different ways the two events can occur is $(m_1)(m_2)$.

Example:

If you have 4 shirts and 5 pairs of pants, 4x5=20, so you have 20 different sets of shirts and pants.

Permutation

A permutation of n different elements in an ordering of the elements such that one element is first, one is second, one is third, and so on. (Order matters)

Number of Permutations of n Elements

The number of permutations of n elements is given by:

$$n*(n-1)*(n-2)*...*3*2*1=n!$$

Example:

The letters in STOP:

(4!)=24 different permutations

Permutations of n Elements Taken r at a Time:

The number of permutations of r elements taken from a set containing n elements with n>r is given by:

$$_nP_r=\frac{n!}{(n-r)!}$$

Example:

Two letters of the word STOP can be arranged in:

$$_4P_2=\frac{4!}{(4-2)!}=12 \text{ different ways}$$

Distinguishable Permutations

Suppose a set of n objects has n_1 of the first object, n_2 of the second object, n_3 of the third object and so on so that:

$$n=n_1+n_2+n_3+...+n_k$$

The number of distinguishable permutations is given by:

$$\frac{n!}{n_1!*n_2!*n_3!*..n_k!}$$

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Example:

Find the number of distinguishable permutations for the letter of BANANA

$$\begin{aligned} \mathrm{B} &\rightarrow n_1 = 1 \\ \mathrm{A} &\rightarrow n_2 = 3 \\ \mathrm{N} &\rightarrow n_3 = 2 \end{aligned} \qquad \frac{6!}{3!*2!*1!} = \frac{720}{12} = 60$$

Combination

A subset where order is NOT important.

Example:

{A, B, C} and {B, A, C} are equivalent

Combinations of n Elements Taken r at a Time

The number of combinations of n elements taken r at a time is given by:

$${}_nC_r = \frac{n!}{(n-r)!*r!} = \frac{{}_nP_r}{r!}$$

Example:

Number of different possible poker hands (5 cards) from a deck of 52 cards.

$${}_{52}C_5 = \frac{52!}{(52-5)!*5!} = \frac{{}_{52}P_5}{5!} = 2{,}598{,}960$$

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