Geometric Sequences Notes

Geometric Sequences

Geometric Sequence - A sequence of terms where the ratios between any consecutive terms are the same.

$a_1,\ a_2,\ a_3,\ a_4,\ldots,a_n$

is geometric if:

$$\frac{a_2}{a_1} = r \quad \frac{a_3}{a_2} = r \quad \frac{a_4}{a_3} = r \quad \frac{a_n}{a_{n-1}} = r$$

such that r≠0

r is the common ratio of the geometric sequence.

Example:

The following sequence is geometric:

$$3, 6, 12, 24, 48...$$

because $\frac{6}{3} = 2, \quad \frac{12}{6} = 2, \quad \frac{24}{12} = 2...$ with the common ratio being 2.

n^{th} Term of a Geometric Sequence

The n^{th} term of a geometric sequence is given by:

$a_n = a_1 r^{n-1} \quad a_1 = a_1 r^0 \quad a_2 = a_1 r^1$

Example:

In the sequence above $(3, 6, 12, 24, 48...)$ find the 15^{th} term:

$n = 15$

$a_{15} = 3(2)^{15-1} = 3(2)^{14} = 49152$

Sum of a Finite Amount of Terms of a Geometric Sequence

Note: this works as long as the common ratio is NOT 1

The sum of the terms from a_1 through a_n is given by:

$$S = a_1\left(\frac{1-r^n}{1-r}\right)$$

Example:

In the sequence above $(3, 6, 12, 24, 48...)$ find the sum of the first 15 terms:

$n = 15$

$$S = 3\left(\frac{1-2^{15}}{1-2}\right) = 98301$$

Sum of an Infinite Geometric Sequence

NOTE: only when the absolute value of the common ratio is LESS than 1
The sum of *all the terms in the sequence* is given by:

$$S = \sum_{n=1}^{\infty} a_1 r^{n-1} = \frac{a_1}{1-r}$$

Example:

The following sequence is geometric:

$5, 2.5, 1.25, 0.625, 0.3125...$

The common ratio is 0.5

The sum of all the terms in the sequence is:

$$S = \sum_{n=1}^{\infty} a_1 r^{n-1} = \frac{a_1}{1-r} = \frac{5}{1-0.5} = 10$$

