

For each of the following, find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$

1. $f(x) = x + 3$

$$g(x) = x^2 - 5$$

$$(f + g)(x) = (x + 3) + (x^2 - 5) = x + 3 + x^2 - 5 = x^2 + x - 2$$

$$(f - g)(x) = (x + 3) - (x^2 - 5) = x + 3 - x^2 + 5 = -x^2 + x + 8$$

$$(fg)(x) = (x + 3)(x^2 - 5) = x^3 - 5x + 3x^2 - 15 = x^3 + 3x^2 - 5x - 15$$

$$\left(\frac{f}{g}\right)(x) = \frac{(x + 3)}{(x^2 - 5)}, \quad x \neq \pm\sqrt{5}$$

2. $f(x) = x^2 + 2x + 1$

$$g(x) = \frac{1}{x}$$

$$(f + g)(x) = (x^2 + 2x + 1) + \left(\frac{1}{x}\right) = x^2 + 2x + 1 + \frac{1}{x}$$

$$(f - g)(x) = (x^2 + 2x + 1) - \left(\frac{1}{x}\right) = x^2 + 2x + 1 - \frac{1}{x}$$

$$(fg)(x) = (x^2 + 2x + 1)\left(\frac{1}{x}\right) = \frac{x^2}{x} + \frac{2x}{x} + \frac{1}{x}x + 2 + \frac{1}{x}$$

$$\left(\frac{f}{g}\right)(x) = \frac{(x^2 + 2x + 1)}{\left(\frac{1}{x}\right)} = (x^2 + 2x + 1) \div \frac{1}{x} = (x^2 + 2x + 1)(x) = x^3 + 2x^2 + x, \quad x \neq 0$$

3. $f(x) = \frac{3x + 2}{x - 5}$

$$g(x) = \sqrt{x - 5}$$

$$(f + g)(x) = \frac{3x + 2}{x - 5} + \sqrt{x - 5}$$

$$(f - g)(x) = \frac{3x + 2}{x - 5} - \sqrt{x - 5}$$

$$(fg)(x) = \frac{3x + 2}{x - 5}(\sqrt{x - 5}) = \frac{(3x + 2)\sqrt{x - 5}}{x - 5}$$

$$\left(\frac{f}{g}\right)(x) = \frac{3x + 2}{x - 5} \div (\sqrt{x - 5}) = \left(\frac{3x + 2}{x - 5}\right)\left(\frac{1}{\sqrt{x - 5}}\right) = \frac{3x + 2}{(x - 5)\sqrt{x - 5}} \times \frac{\sqrt{x - 5}}{\sqrt{x - 5}} = \frac{(3x + 2)\sqrt{x - 5}}{(x - 5)^2}$$

$$4. f(x) = \frac{2x-5}{3x+7}$$

$$g(x) = \frac{5x-4}{2x-7}$$

$$\begin{aligned} (f+g)(x) &= \frac{2x-5}{3x+7} + \frac{5x-4}{2x-7} = \frac{(2x-5)(2x-7)}{(3x+7)(2x-7)} + \frac{(5x-4)(3x+7)}{(3x+7)(2x-7)} \\ &= \frac{4x^2 - 14x - 10x + 35}{6x^2 - 21x + 14x - 49} + \frac{15x^2 + 35x - 12x - 28}{6x^2 - 21x + 14x - 49} \\ &= \frac{4x^2 - 24x + 35 + 15x^2 + 23x - 28}{6x^2 - 7x - 49} = \frac{19x^2 - x + 7}{6x^2 - 7x - 49} \end{aligned}$$

$$\begin{aligned} (f-g)(x) &= \frac{2x-5}{3x+7} - \frac{5x-4}{2x-7} = \frac{(2x-5)(2x-7)}{(3x+7)(2x-7)} - \frac{(5x-4)(3x+7)}{(3x+7)(2x-7)} \\ &= \frac{4x^2 - 14x - 10x + 35}{6x^2 - 21x + 14x - 49} - \frac{15x^2 + 35x - 12x - 28}{6x^2 - 21x + 14x - 49} \\ &= \frac{4x^2 - 24x + 35 - (15x^2 + 23x - 28)}{6x^2 - 7x - 49} = \frac{-11x^2 - 47x + 63}{6x^2 - 7x - 49} \end{aligned}$$

$$(fg)(x) = \left(\frac{2x-5}{3x+7}\right)\left(\frac{5x-4}{2x-7}\right) = \frac{(2x-5)(5x-4)}{(3x+7)(2x-7)} = \frac{10x^2 - 8x - 25x + 20}{6x^2 - 21x + 14x - 49} = \frac{10x^2 - 33x + 20}{6x^2 - 7x - 49}$$

$$\begin{aligned} \left(\frac{f}{g}\right)(x) &= \left(\frac{2x-5}{3x+7}\right) \div \left(\frac{5x-4}{2x-7}\right) = \left(\frac{2x-5}{3x+7}\right)\left(\frac{2x-7}{5x-4}\right) \\ &= \frac{(2x-5)(2x-7)}{(3x+7)(5x-4)} = \frac{4x^2 - 14x - 10x + 35}{15x^2 - 12x + 35x - 28} = \frac{4x^2 - 24x + 35}{15x^2 + 23x - 28} \end{aligned}$$