Factor each of the following (if possible).

1. $x^2 - 2x - 15$

$D = B^2 - 4AC$

$D = (-2)^2 - 4(1)(-15)$

$D = 4 - 60 = 64$

FACTORABLE!

	x^2	
		-15

Multiply to: -15

Add to: -2

-5 and 3

	x^2	$-5x$
	$3x$	-15

Find the GCF in all directions

	x	-5
x	x^2	$-5x$
3	$3x$	-15

$x^2 - 2x - 15 = (x+3)(x-5)$

Check:

$(x+3)(x-5)$

$x^2 - 5x + 3x - 15$

$x^2 - 2x - 15$

2. $2x^2 + 14x + 24$

Common 2:

$2(x^2 + 7x + 12)$

$D = B^2 - 4AC$

$D = (7)^2 - 4(1)(12)$

$D = 49 - 48 = 1$

FACTORABLE!

	x^2	
		12

Multiply to: 12

Add to: 7

3 and 4

	x^2	$3x$
	$4x$	12

Find the GCF in all directions

	x	3
x	x^2	$3x$
4	$4x$	12

$= 2x^2 + 14x + 24 = 2(x+3)(x+4)$

Check:

$2(x+3)(x+4)$

$= 2(x^2 + 4x + 3x + 12)$

$= 2(x^2 + 7x + 12)$

$= 2x^2 + 14x + 24$

3. $4x^2+14x-60$

Common 2:

$2\left(2x^2+7x-30\right)$

$D=B^2-4AC$

$D=(7)^2-4(2)(-30)$

$D=49+240=289$

$17^2=289$

FACTORABLE!

	$2x^2$	
		-30

Multiply to: -60

Add to: 7

-5 and 12

	$2x^2$	$-5x$
	$12x$	-30

Find the GCF in all directions

	$2x$	-5
x	$2x^2$	$-5x$
6	$12x$	-30

$4x^2+14x-60=2(x+6)(2x-5)$

Check:

$2(x+6)(2x-5)$

$=2\left(2x^2-5x+12x-30\right)$

$=2\left(2x^2+7x-30\right)$

$=4x^2+14x-60$

4. $2x^2+6x-1$

$D=B^2-4AC$

$D=(6)^2-4(2)(-1)$

$D=36+8=44$

NOT FACTORABLE!

5. $6x^2+17x-14$

$D=B^2-4AC$

$D=(17)^2-4(6)(-14)$

$D=289+336=625$

$25^2=625$

FACTORABLE!

	$6x^2$	
		-14

Multiply to: -84

Add to: 17

-4 and 21

	$6x^2$	$-4x$
	$21x$	-14

Find the GCF in all directions

	$3x$	-2
$2x$	$6x^2$	$-4x$
7	$21x$	-14

$6x^2+17-14=(3x-2)(2x+7)$

Check:

$(3x-2)(2x+7)$

$=\left(6x^2+21x-4x-14\right)$

$=\left(6x^2+17x-14\right)$

Solve each of the following by factoring.

6. $x^2 - 3x - 10 = 0$

$D = B^2 - 4AC$

$D = (-3)^2 - 4(1)(-10)$

$D = 9 + 40 = 49$

FACTORABLE!

	x^2	
		-10

Multiply to: -10

Add to: -3

-5 and 2

	x^2	$-5x$
	$2x$	-10

Find the GCF in all directions

	x	-5
x	x^2	$-5x$
2	$2x$	-10

$x^2 - 3x - 10 = (x+2)(x-5) = 0$

Check:

$(x+2)(x-5) = 0$

$x^2 - 5x + 2x - 10 = 0$

$x^2 - 3x - 10 = 0$

Roots:

$x + 2 = 0 \quad or \quad x - 5 = 0$

$x = -2 \qquad\qquad x = 5$

Roots : $\{x \mid x = -2, 5\}$

7. $2x^2 + 6x - 8 = 0$

Common 2:

$2(x^2 + 3x - 4) = 0$

$D = B^2 - 4AC$

$D = (3)^2 - 4(1)(-4)$

$D = 9 + 16 = 25$

FACTORABLE!

	x^2	
		-4

Multiply to: -4

Add to: 3

-1 and 4

	x^2	$-x$
	$4x$	-4

Find the GCF in all directions

	x	-1
x	x^2	$-x$
4	$4x$	-4

$2x^2 + 6x - 8 = 2(x+4)(x-1) = 0$

Check:

$2(x+4)(x-1) = 0$

$2(x^2 - x + 4x - 4) = 0$

$2(x^2 + 3x - 4) = 0$

$2x^2 + 6x - 8 = 0$

Roots:

$x + 4 = 0 \quad or \quad x - 1 = 0$

$x = -4 \qquad\qquad x = 1$

Roots : $\{x \mid x = -4, 1\}$

8. $6x^2+27x-15=0$

Common 3:

$3\left(2x^2+9x-5\right)=0$

$D=B^2-4AC$

$D=(9)^2-4(2)(-5)$

$D=81+40=121$

FACTORABLE!

	$2x^2$	
		-5

Multiply to: -10
Add to: 9
-1 and 10

	$2x^2$	$-x$
	$10x$	-5

Find the GCF in all directions

	$2x$	-1
x	$2x^2$	$-x$
5	$10x$	-5

$6x^2+27x-15=3(2x-1)(x+5)=0$

Check:

$3(2x-1)(x+5)=0$

$3\left(2x^2+10x-x-5\right)=0$

$3\left(2x^2+9x-5\right)=0$

$6x^2+27x-15=0$

Roots:

$2x-1=0$ *or* $x+5=0$

$2x=1$ $x=-5$

$x=\frac{1}{2}$

Roots : $\left\{x\middle|x=-5,\frac{1}{2}\right\}$

9. $2x^2-11x+15=0$

$D=B^2-4AC$

$D=(-11)^2-4(2)(15)$

$D=121+120=1$

FACTORABLE!

	$2x^2$	
		15

Multiply to: 30
Add to: -11
-5 and -6

	$2x^2$	$-5x$
	$-6x$	15

Find the GCF in all directions

	$2x$	-5
x	$2x^2$	$-5x$
-3	$-6x$	15

$2x^2-11x+15=(2x-5)(x-3)=0$

Check:

$(2x-5)(x-3)=0$

$2x^2-6x-5x+15=0$

$2x^2-11x+15=0$

Roots:

$2x-5=0$ *or* $x-3=0$

$2x=5$ $x=3$

$x=\frac{5}{2}=2\frac{1}{2}$

Roots : $\left\{x\middle|x=2\frac{1}{2},3\right\}$

10. $105x^2 - 215x + 100 = 0$

Common 5:

$5\left(21x^2 - 43x + 20\right) = 0$

$D = B^2 - 4AC$

$D = (-43)^2 - 4(21)(20)$

$D = 1849 - 1680 = 169$

$13^2 = 169$

FACTORABLE!

	$21x^2$	
		20

Multiply to: 420

Add to: -43

-28 and -15

	$21x^2$	$-28x$
	$-15x$	20

Find the GCF in all directions

	$3x$	-4
$7x$	$21x^2$	$-28x$
-5	$-15x$	20

$105x^2 - 215x + 100 = 5(3x - 4)(7x - 5) = 0$

Check:

$5(3x - 4)(7x - 5) = 0$

$5\left(21x^2 - 15x - 28x + 20\right) = 0$

$5\left(21x^2 - 43x + 20\right) = 0$

$105x^2 - 215x + 100 = 0$

Roots:

$3x - 4 = 0$ *or* $7x - 5 = 0$

$3x = 4$ $\quad$ $7x = 5$

$x = \frac{4}{3} = 1\frac{1}{3}$ $\quad$ $x = \frac{5}{7}$

Roots : $\left\{x \middle| x = \frac{5}{7}, 1\frac{1}{3}\right\}$

Graph each of the following. Also, put each into vertex form, list all intercepts, and find the roots by factoring (if possible).

11. $f(x)=2x^2+4x+6$

$A=2, B=4, C=6$

$D=B^2-4AC$

$D=(4)^2-4(2)(6)$

$D=16-48=-32$

NOT FACTORABLE!

COMMON FACTOR OF 2

$f(x)=2\left(x^2+2x+3\right)$

Vertex : $x=\frac{-B}{2A}=\frac{-(4)}{2(2)}-\frac{-4}{4}=-1$

$f(-1)=2\left((-1)^2+2(-1)+3\right)=2(2)=4$

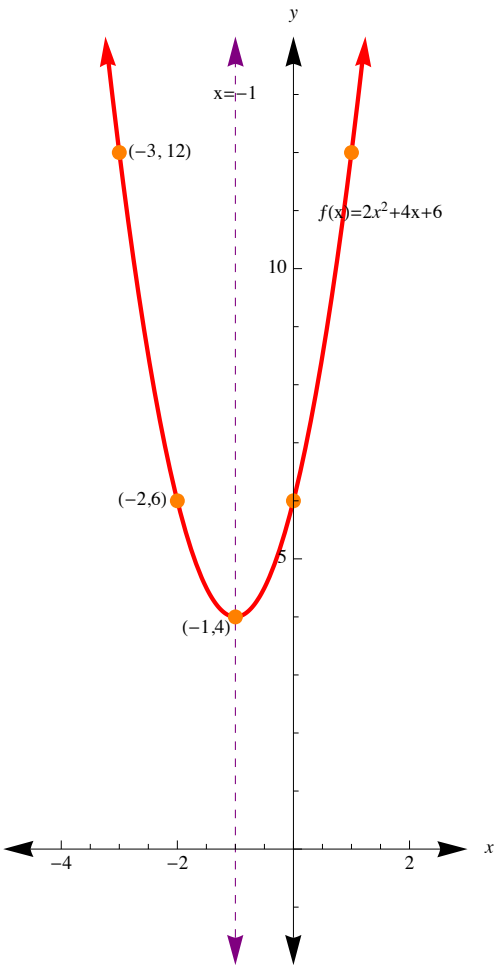
Vertex : $(-1,4)$

x	y	(x,y)
-3	12	$(-3,12)$
-2	6	$(-2,6)$
-1	4	$(-1,4)$
0	6	$(0,6)$
1	12	$(1,12)$

Since the discriminant is negative, there are zero real roots.

Axis of symmetry : $x=-1$

$y-\text{int}:(0,6)$



12. $f(x)=3x^2+2x-8$

$A=3, B=2, C=-8$

$D=B^2-4AC$

$D=(2)^2-4(3)(-8)$

$D=4+96=100$

FACTORABLE!

	$3x^2$	
		-8

Multiply to: -24

Add to: 2

-4 and 6

	$3x^2$	$-4x$
	$6x$	-8

Find the GCF in all directions

	$3x$	-4
x	$3x^2$	$-4x$
2	$6x$	-8

$f(x)=3x^2+2x-8=(3x-4)(x+2)$

Check:

$(3x-4)(x+2)$

$=3x^2+6x-4x-8$

$=3x^2+2x-8$

Roots:

$3x-4=0$ or $x+2=0$

$3x=4$ $x=-2$

$x=\frac{4}{3}=1\frac{1}{3}$

$\left(1\frac{1}{3},0\right)$ or $(-2,0)$

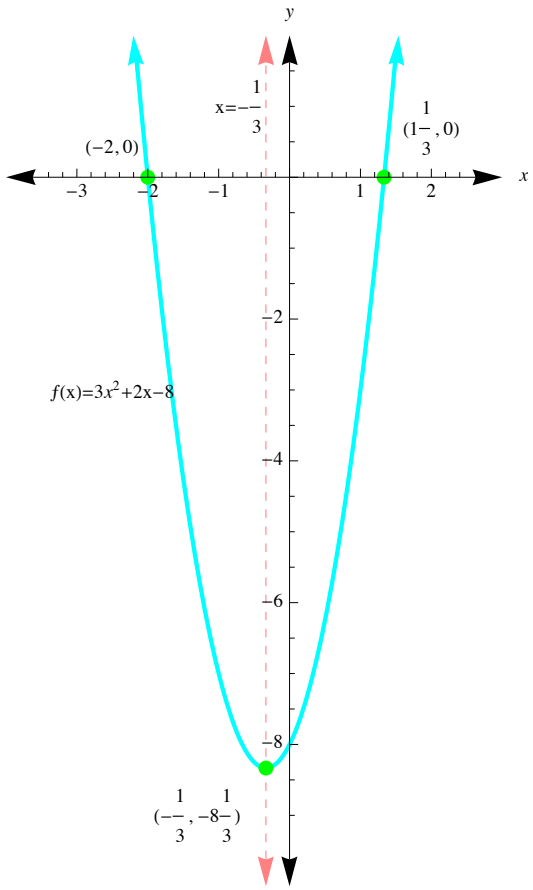
Roots : $\left\{x\middle|x=1\frac{1}{3},-2\right\}$

Vertex : $x=\frac{-B}{2A}=\frac{-(2)}{2(3)}=\frac{-2}{6}=-\frac{1}{3}$

$f(x)=3\left(-\frac{1}{3}\right)^2+2\left(-\frac{1}{3}\right)-8=-8\frac{1}{3}$

Vertex : $\left(-\frac{1}{3},-8\frac{1}{3}\right)$

Axis of Symmetry : $x=-\frac{1}{3}$



13. $f(x) = x^2 + 2x - 4$

$A = 1, B = 2, C = -4$

$D = B^2 - 4AC$

$D = (4)^2 - 4(1)(-4)$

$D = 4 + 16 = 20$

NOT FACTORABLE!

Vertex : $x = \frac{-B}{2A} = \frac{-(2)}{2(1)} - \frac{-2}{2} = -1$

$f(-1) = (-1)^2 + 2(-1) - 4 = -5$

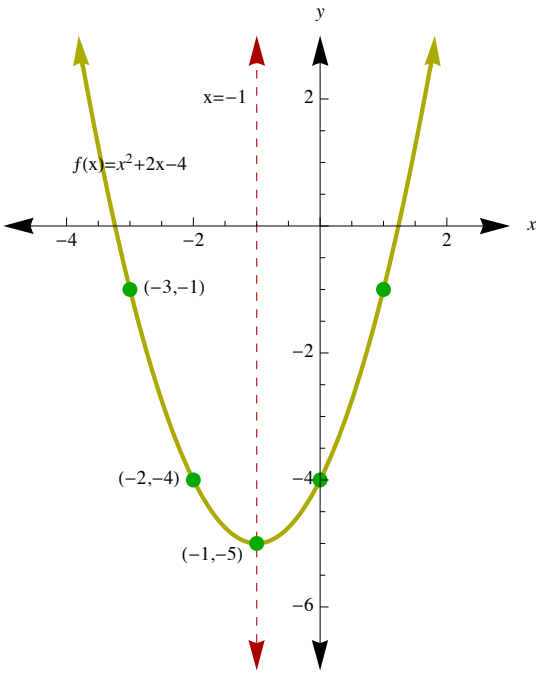
Vertex : $(-1, -5)$

x	y	(x, y)
-3	-1	$(-3, -1)$
-2	-4	$(-2, -4)$
-1	-5	$(-1, -5)$
0	-4	$(0, -4)$
1	-1	$(1, -1)$

Since the discriminant is positive, there are two real roots.

Axis of symmetry : $x = -1$

y – int : $(0, -4)$



14. $f(x)=9x^2-49$

$A=9, B=0, C=-49$

$D=B^2-4AC$

$D=(0)^2-4(9)(-49)$

$D=0+1764=1764$

$\sqrt{1764}=42$

FACTORABLE!

	$9x^2$	
		-49

Multiply to: -441

Add to: 0

-21 and 21

	$9x^2$	$-21x$
	$21x$	-49

Find the GCF in all directions

	$3x$	-7
$3x$	$9x^2$	$-21x$
7	$21x$	-49

$f(x)=(3x+7)(3x-7)$

Check:

$f(x)=(3x+7)(3x-7)$

$f(x)=9x^2-21x+21x-49$

$f(x)=9x^2-49$

Roots:

$3x+7=0$ or $3x-7=0$

$3x=-7$ $\quad$ $3x=7$

$x=-\frac{7}{3}=-2\frac{1}{3}$ $\quad$ $x=\frac{7}{3}=2\frac{1}{3}$

$\left(-2\frac{1}{3},0\right)$ or $\left(2\frac{1}{3},0\right)$

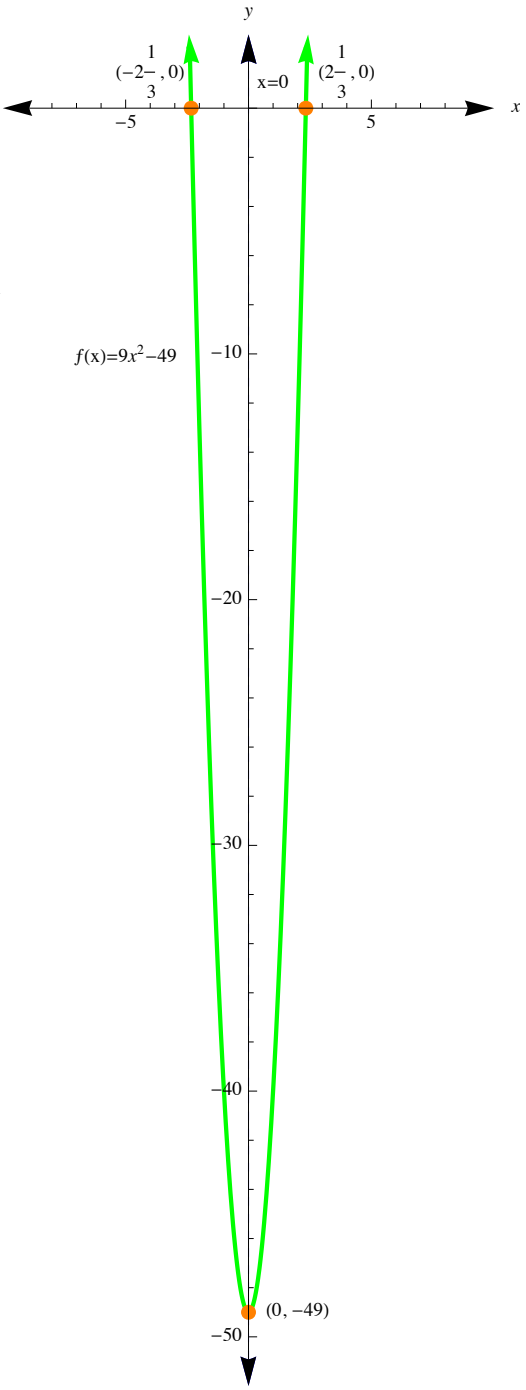
Roots : $\left\{x\middle|x=-2\frac{1}{3},2\frac{1}{3}\right\}$

Vertex : $x=\frac{-B}{2A}=\frac{-(0)}{2(9)}=\frac{0}{18}=0$

$f(x)=9(0)^2-49=-49$

Vertex : $(0,-49)$

Axis of Symmetry : $x=0$



15. $f(x)=x^2-3x-10$

$A=1, B=-3, C=-10$

$D=B^2-4AC$

$D=(-3)^2-4(1)(-10)$

$D=9+40=49$

FACTORABLE!

	x^2	
		-10

Multiply to: -10

Add to: -3

-5 and 2

	x^2	$-5x$
	$2x$	-10

Find the GCF in all directions

	x	-5
x	x^2	$-5x$
2	$2x$	-10

$f(x)=(x-5)(x+2)$

Check:

$f(x)=(x-5)(x+2)$

$f(x)=x^2+2x-5x-10$

$f(x)=x^2-3x-10$

Roots:

$x-5=0$ or $x+2=0$

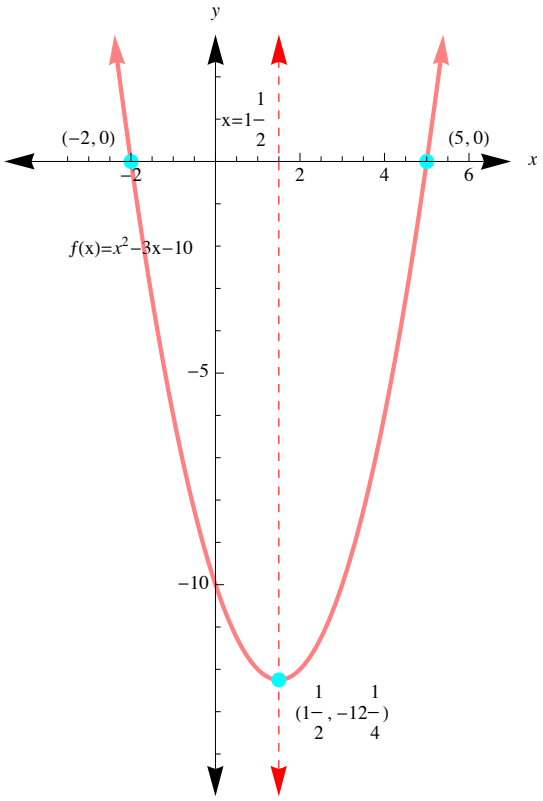
$x=5$ $x=-2$

$(5,0)$ or $(-2,0)$

Roots : $\{x|x=-2,5\}$

Vertex : $x=\frac{-B}{2A}=\frac{-(-3)}{2(1)}=\frac{3}{2}=1\frac{1}{2}$

$f\left(\frac{3}{2}\right)=\left(\frac{3}{2}\right)^2-3\left(\frac{3}{2}\right)-10$



Work with fractions:

$\left(\frac{3}{2}\right)^2=\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)=\frac{9}{4}$

$-3\left(\frac{3}{2}\right)=-\frac{3}{1}\left(\frac{3}{2}\right)=-\frac{9}{2}$

$f\left(\frac{3}{2}\right)=\frac{9}{4}-\frac{9}{2}-10$

$\frac{9}{4}-\frac{9}{2}=\frac{(9)(2)-(4)(9)}{(4)(2)}=\frac{18-36}{8}=-\frac{18}{8}=-\frac{9}{4}$

$-\frac{9}{4}-10=-\frac{9}{4}-\frac{10}{1}=-\frac{(9)(1)+(4)(10)}{(4)(1)}$

$=-\frac{9+40}{4}=-\frac{49}{4}=-12\frac{1}{4}$

$f\left(\frac{3}{2}\right)=-12\frac{1}{4}$

Vertex : $\left(1\frac{1}{2},-12\frac{1}{4}\right)$

Axis of Symmetry : $x=1\frac{1}{2}$

